MENSURATION

Definition

1. Mensuration: It is a branch of mathematics which deals with the lengths of lines, areas of surfaces and volumes of solids.
2. Plane Mensuration: It deals with the sides, perimeters and areas of plane figures of different shapes.
3. Solid Mensuration: It deals with the areas and volumes of solid objects.

Important Formulae

Right Angled Triangle:

\[(AC)^2 = (AB)^2 + (BC)^2\]

or, \[h^2 = p^2 + b^2\]

If \(AC = 5m, AB = 4m\) then

\[(BC)^2 = (AC)^2 - (AB)^2\]

\[= 25 - 16 = 9\]

\[\therefore BC = 3m\]

Rectangle: A rectangle is a plane

Whose opposite sides are equal and diagonals are equal. Each angle is equal to \(90^\circ\).

Here \(AB = CD;\) length \(l = 4m\)

\(AD = BC;\) breadth \(b = 3m\)

1. Perimeter of a rectangle = \(2(\text{length} + \text{breadth})\)

\[= 2(l + b)\]

\[= 2(4 + 3) = 14\ m\]

2. Area of rectangle = \(\text{length} \times \text{breadth} = l \times b = 4 \times 3\)

\[= 12\ m^2\]
3. Length of a rectangle : \[ \frac{\text{area}}{\text{breadth}} = \frac{A}{b} = \frac{12}{3} = 4 \text{ m} \]
   
or, \[ \left( \frac{\text{perimeter}}{2} - \text{breadth} \right) = \left( \frac{14}{2} - 3 \right) = 4 \text{ m} \]

   Breadth of a rectangle : \[ \frac{\text{area}}{\text{length}} = \frac{A}{l} = \frac{12}{4} = 3 \text{ m} \]
   
or, \[ \left( \frac{\text{perimeter}}{2} - \text{length} \right) = \left( \frac{14}{2} - 4 \right) = 3 \text{ m} \]

4. Diagonal of rectangle : \[ \sqrt{(\text{length})^2 + (\text{breadth})^2} \]
   
   \[ = \sqrt{l^2 + b^2} = \sqrt{4^2 + 3^2} \]
   
   \[ = \sqrt{16 + 9} = \sqrt{25} = 5 \text{ m} \]

**Square** : A square is a plane figure bounded by four equal sides having all its angle as right angles.

Here \( AB = BC = CD = AD = 5 \text{ m} = a \text{(Let)} \)

1. Perimeter of square = \( 4 \times \text{sides} = 4a \)
   
   \[ = 4 \times 5 = 20 \text{ m} \]

2. Area of a square = \( (\text{sides})^2 = a^2 = (5)^2 = 25 \text{ sq. m} \)

3. Side of a square = \( \sqrt{\text{area}} = \sqrt{25} = 5 \text{ m} \) or, \[ \frac{\text{Perimeter}}{4} = \frac{20}{4} = 5 \text{ m} \]

4. Diagonal of a square = \( \sqrt{2} \times \text{side} = \sqrt{2} \times a \)
   
   \[ = \sqrt{2} \times 5 = 5\sqrt{2} \text{ m} \]

5. Side of a square = \( \frac{\text{diagonal}}{\sqrt{2}} = \frac{5\sqrt{2}}{\sqrt{2}} = 5 \text{ m} \)

**Triangle** :

1. Area of triangle = \( \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times b \times h \)
   
   \[ = \frac{1}{2} \times 15 \times 12 = 90 \text{ sq. cm} \]

   Here \( AD = 12 \text{ cm} = \text{height}, BC = 15 \text{ cm} = \text{base} \)
2. Semi perimeter of a triangle
\[ S = \frac{a+b+c}{2} = \frac{10+8+6}{2} = 12 \text{ cm} \]
here BC = a, AC = b, AB = c

3. Area of triangle = \( \sqrt{s(s-a)(s-b)(s-c)} \)
where \( a = 10 \text{ cm}, b = 8 \text{ cm}, c = 6 \text{ cm}, s = 12 \text{ cm} \)
\[ = \sqrt{12(12-10)(12-8)(12-6)} \]
\[ = \sqrt{12 \times 2 \times 4 \times 6} = 24 \text{ cm}^2 \]

4. Perimeter of a triangle = \( 2s = (a + b + c) \)
\[ = 10 + 8 + 6 = 24 \text{ cm} \]

5. Area of an equilateral triangle = \( \frac{\sqrt{3}}{4} \times (\text{side})^2 \)
\[ = \frac{\sqrt{3}}{4} \times (4\sqrt{3})^2 \]
\[ = \frac{\sqrt{3}}{4} \times 48 = 12\sqrt{3} \text{ cm}^2 \]

6. Height of an equilateral triangle = \( \frac{\sqrt{3}}{2} \times (\text{side}) = \frac{\sqrt{3}}{2} \times 4\sqrt{3} \)
\[ = 6 \text{ cm} \]

7. Perimeter of an equilateral triangle = \( 3 \times (\text{side}) \)
\[ = 3 \times 4\sqrt{3} = 12\sqrt{3} \text{ cm} \]

Quadrilateral:
Parallelogram:
(i) Area of parallelogram = base \times height
\[ = b \times h \]
\[ = 8 \times 5 = 40 \text{ sq.cm.} \]
(ii) Perimeter of a parallelogram = \( 2(AB + BC) \)
\[
2(8 + 5) = 26 \text{ cm}
\]

**Rhombus:**

(i) Area of rhombus \(= \frac{1}{2} \times \text{(product of diagonals)}\)
\[
= \frac{1}{2} \times (d_1 \cdot d_2) = \frac{1}{2} \times 8 \times 6 = 24 \text{ cm}^2
\]

(ii) Perimeter of rhombus = 4 x side = 4a
here AB = BC = CD = AD = 4a
AC = d_1, BD = d_2

**Trapezium:**

(i) Area of a trapezium \(= \frac{1}{2} \times \text{(sum of parallel sides)} \times \text{height}\)
\[
= \frac{1}{2} \times (a + b) \times h = \frac{1}{2} \times (15 + 17) \times 10
\]
\[
= \frac{1}{2} \times 32 \times 10 = 160 \text{ cm}^2
\]

**Regular Hexagon:**

(i) Area of a regular hexagon \(= 6 \times \frac{\sqrt{3}}{4} \times \text{(side)}^2\)
(ii) Perimeter of a regular hexagon = 6 x (side)

**Circle:**

(i) Circumference of a circle \(= \pi \times \text{diameter}\)
\[
= \pi \times 2r = 2\pi r
\]
= 2 × \( \frac{22}{7} \times 42 = 264 \) cm

(ii) Radius of a circle = \( \frac{\text{circumference}}{2 \pi} = \frac{264 \times 7}{2 \times 22} = 42 \) cm

(iii) Area of a circle = \( \pi \times r^2 = \frac{22}{7} \times 42^2 = \frac{22}{7} \times 42 \times 42 = 5544 \) cm\(^2\)

(iv) Radius of a circle = \( \sqrt{\frac{\text{area}}{\pi}} \)

\[ = \sqrt{\frac{5544}{22}} \times 7 = \sqrt{1764} = 42 \) cm

(v) Area of a semi circle = \( \frac{1}{2} \pi r^2 = \frac{1}{8} \pi d^2 \)

\[ = \frac{1}{2} \times \frac{22}{7} \times 42^2 = 2772 \) cm\(^2\)

(vi) Circumference of semi circle = \( \frac{22}{7} \times 42 = 132 \) cm

(vii) Perimeter of semi circle = \( (\pi r + 2r) = (\pi + 2) r = (\pi + 2) \frac{d}{2} \)

(viii) Area of sector OAB = \( \frac{x}{360} \times \pi r^2 \)

(x being the central angle)

\[ = \frac{30^\circ}{360^\circ} \times \frac{22}{7} \times 3.5 \times 3.5 = 3.21 \) sq. m.

(ix) Central angle by arc AB = \( 360^\circ \times \frac{\text{area of OAB}}{\text{area of circle}} \)

\[ = \frac{360^\circ}{\frac{22}{7} \times 3.5 \times 3.5} \times \frac{360 \times 321}{22 \times 35 \times 5} = 30^\circ \) (approx)

(x) Radius of circle = \( \sqrt{\frac{360^\circ}{\text{central angle by arc}}} \times \frac{\text{area of OAB}}{\pi} \)

\[ = \sqrt{\frac{360^\circ}{30^\circ} \times \frac{3.21}{\frac{22}{7}}} = \sqrt{\frac{134.82}{11}} = \sqrt{12.23} = 3.5 \) m.

(xi) Area of ring

= difference of the area of two circle

\[ = 4 \times 3 = 12 \) cm\(^2\)
\[ \pi R^2 - \pi r^2 = (R^2 - r^2) = \pi (R + r)(R - r) = (\text{sum of radius})(\text{diff. of radius}) \]
\[ = \frac{22}{7} \times (4 + 3)(4 - 3) = \frac{22}{7} \times 7 \times 1 \]
\[ = 22 \text{ sq. cm.} \]

**Cuboid and Cube :**

(i) Total surface area of cuboid
\[ = 2(lb + bh + hl) \text{ sq. unit} \]
Here \( l = \text{length}, b = \text{breadth}, h = \text{height} \)
\[ = 2(12 \times 8 + 8 \times 6 + 6 \times 12) \]
\[ = 2(96 + 48 + 72) = 2 \times 216 = 432 \text{ sq. cm.} \]

(ii) Volume of a cuboid \( = (\text{length} \times \text{breadth} \times \text{height}) = lbh \)
\[ = 12 \times 8 \times 6 = 576 \text{ cubic cm} \]

(iii) Diagonal of a cuboid
\[ = \sqrt{l^2 + b^2 + h^2} = \sqrt{12^2 + 8^2 + 6^2} \]
\[ = \sqrt{144 + 64 + 36} = \sqrt{244} = 2\sqrt{61} \text{ cm.} \]

(iv) Length of cuboid \[ = \frac{V}{B \times H} = \frac{v}{b \times h} \]

(v) Breadth of cuboid \[ = \frac{V}{L \times H} = \frac{v}{l \times h} \]

(vi) Height of cuboid \[ = \frac{V}{L \times B} = \frac{v}{l \times b} \]

(vii) Volume of cube \( = (\text{side})^3 \)
\[ = 12^3 \]
\[ = 1728 \text{ cubic cm} \]

\[ \text{Cube : All sides are equal} = 12 \text{ cm} \]

(viii) Sides of a cube \[ = \sqrt[3]{\text{Volume}} \]
\[ = \sqrt[3]{1728} = 12 \text{ cm} \]
(ix) Diagonal of cube = $\sqrt{3} \times \text{(side)} = \sqrt{3} \times 12 = 12\sqrt{3}$ cm
(x) Total surface area of a cube = $6 \times \text{(side)}^2 = 6 \times 12^2 = 864$ sq.cm

**Right Circular Cylinder:**

(i) Area of curved surface

\[ = (\text{perimeter of base}) \times \text{height} \]
\[ = 2\pi rh \text{ sq. unit} \]
\[ = 2 \times \frac{22}{7} \times 7 \times 15 = 660 \text{ sq. cm} \]

(ii) Total surface area = area of circular ends + curved surface area

\[ = 2\pi r^2 + 2\pi rh = 2\pi r(r + h) \text{ sq. unit} \]
\[ = 2 \times \frac{22}{7} \times 7(15 + 7) \]
\[ = 2 \times 22 \times 22 \]
\[ = 968 \text{ sq. cm.} \]

(iii) Volume = (area of base) \times \text{height}

\[ = (\pi r^2) \times h = \pi r^2h \]
\[ = \frac{22}{7} \times 7 \times 7 \times 15 = 2310 \text{ cubic cm.} \]

(iv) Volume of a hollow cylinder = $\pi R^2h - \pi r^2h$

\[ = \pi h(R^2 - r^2) = \pi h (R + r)(R - r) \]

Here $R, r$ are outer and inner radii respectively and $h$ is the height.

**Cone:**

(i) In right angled $\triangle OAC$, we have

\[ l^2 = h^2 + r^2 \]

(Here $r = 35 \text{ cm}$, $l = 37 \text{ cm}$, $h = 12 \text{ cm}$)
Or, \( l = \sqrt{h^2 + r^2} \)

\[ h = \sqrt{l^2 - r^2}, \quad r = \sqrt{l^2 - h^2} \]

where \( l \) = slant height, \( h \) = height, \( r \) = radius of base

(ii) Curved surface area = \( \frac{1}{2} \times \) (perimeter of base) \( \times \) slant height

\[ = \frac{1}{2} \times 2\pi r \times l = \pi rl \text{ sq. unit} \]

\[ = \frac{22}{7} \times 35 \times 37 = 4070 \text{ sq. cm} \]

(iii) Total surface area \( S = \) area of circular base + curved surface area

\[ = (\pi r^2 + \pi rl) = \pi r(r + l) \text{ sq. unit} \]

\[ = \frac{22}{7} \times 35(37 + 35) = 7920 \text{ sq. cm} \]

(iv) Volume of cone = \( \frac{1}{3} \) (area of base) \( \times \) height

\[ = \frac{1}{3} (\pi r^2) \times h = \frac{1}{3} \pi r^2 h \text{ cubic unit} \]

\[ = \frac{1}{3} \times \frac{22}{7} \times 35 \times 35 \times 12 \]

\[ = 15400 \text{ cubic cm} \]

**Frustum of Cone :**

(v) Volume of frustum = \( \frac{1}{3} \pi h(R^2 + r^2 + Rr) \) cubic unit

(vi) Lateral surface = \( \pi l(R + r) \)

where \( l^2 = h^2 + (R - r)^2 \)

(vii) Total surface area = \( \pi [R^2 + r^2 + l(R + r)] \)

\( R, \ r \) be the radius of base and top the frustum

ABB’A’ \( h \) and \( l \) be the vertical height and slant height respectively.

**Sphere :**
(i) Surface area = \(4\pi r^2\)

\[= 4 \times \frac{22}{7} \times (10.5)^2 = 1386 \text{ sq. cm}\]

Here, \(d = 21 \text{ cm} \implies r = 10.5 \text{ cm}\)

(ii) Radius of sphere = \(\sqrt{\frac{Surface\ area}{4\pi}} = \sqrt{\frac{1386 \times 7}{4 \times 22}} = 10.5 \text{ cm}\)

(iii) Diameter of sphere = \(\sqrt{\frac{Surface}{4\pi}} = \sqrt{\frac{1386 \times 7}{22}} = 21 \text{ cm}\)

(iv) Volume of sphere \(V = \frac{4}{3} \pi r^3 = \frac{4}{3} \pi \left(\frac{d}{2}\right)^3 = \frac{1}{6} \pi d^3\)

\[= \frac{1}{6} \times \frac{22}{7} \times 21 \times 21 \times 21 = 4831 \text{ cubic cm}\]

(v) Radius of sphere = \(\sqrt{\frac{3}{4\pi} \times Volume\ of\ sphere}\)

(vi) Diameter = \(\sqrt{\frac{3 \times 6 \times V}{\pi}}\)

(vii) Volume of spherical ring = \(\frac{4}{3} \pi (R^3 - r^3)\)

(viii) Curved surface of hemisphere = \(2\pi r^2\)

(ix) Volume of hemisphere = \(\frac{2}{3} \pi r^3\)

(x) Total surface area of hemisphere = \(3\pi r^2\)

**Note**: \(V = \) volume, \(A = \) area, \(h = \) height, \(b = \) base, breadth, \(d = \) diameter, \(R = \) outer radius, \(r = \) inner radius, \(\pi = \frac{22}{7} = 3.142, a = \) side.

### Prism and Pyramid

#### Prism

1. **Solid**: Bodies which have three dimensions in space are called solid. For example, a block of wood.

   A body, which has the three dimensions length, breadth and height, is a solid, whereas a rectangle with its two dimensions (length and breadth) is not a solid.

2. **Prism**: A prism is a solid, bounded by plane faces of which two opposite sides known as bases are parallel and congruent polygons.

3. **Base**: The congruent and parallel faces of a
prism are called its bases.
The other faces of a prism can be either
oblique to the faces or perpendicular
to them.

4. **Right prism**: A right prism is a prism in
which lateral sides are rectangular or
perpendicular to their bases.

5. **Lateral faces**: The side faces of a prism are called its lateral faces.

6. **Lateral surface area**: The area of all the lateral faces of a prism is called its
lateral surface area.

**Note**: In a right prism having polygons of n sides as bases.

(i) the number of vertices = 2
(ii) the number of edges = 3n
(iii) the number of lateral faces = (n + 1), and
(iv) all the lateral faces are rectangular.

**Formulae**

| (i) | Volume of a right prism = (Area of its base) x height |
| (ii) | Lateral surface area of a right prism |
|      | =(perimeter of its base) x height |
| (iii) | Total surface area of a right prism |
|      | =(lateral surface area) + 2(area of the base) |

**Pyramid**

1. **Pyramid**: A solid of triangular lateral
sides having a common vertex and
plane rectilinear bases with equal
sides is called pyramid.

2. **Height of the pyramid**: The length
of perpendicular drawn from the vertex
of a pyramid to its base is called the
height of the pyramid.
The side faces of pyramid form its lateral surface.

3. **Regular pyramid**: If the base of a pyramid is a regular figure i.e., a polygon with all sides equal and all angles equal, then it is called a regular pyramid.

4. **Right pyramid**: If the foot of the perpendicular from the vertex of a pyramid to its base is the centre of the base then it is called a right pyramid.

5. **Slant height of a regular right pyramid**: The slant height of a regular right pyramid is the length of the line segment joining the vertex to the mid-point of one of the sides of the base.

6. **Tetrahedron**: When the base of a right pyramid is a triangle, then it is called a tetrahedron.

7. **Regular tetrahedron**: A right pyramid with equilateral triangle as its base is called a regular tetrahedron.